The Effect of Gap Size on Dipole Impedance Using the Induced EMF Method

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***Abstract*—The dipole is the fundamental elemental antenna. Moreover, the electric dipole and its monopole equivalent on a groundplane are widely used in practice. Despite the long history of dipole research, its complete impedance behaviour remains elusive. In numerical techniques, such as the method of moments, a gap voltage feed can be expected to give a well-defined radiation conductance but a susceptance which is dissimilar to that of a realized antenna, whereas an impressed current feed can give a well-defined radiation resistance, but dissimilar reactance. The reason is that neither of these feeds accurately model the input region of a practical dipole. Two analytic approaches to the dipole impedance are available - the wave structure method and the induced EMF method. The wave structure method does not lend itself to feed detail, but reveals the impact of dipole thickness and length on the impedance of dipoles which is not available from any other approach. It is reliable for short lengths but it remains restricted to an infinitesimal feed gap, i.e., different to a practical dipole antenna. The induced EMF method is accurate for short and impracticably thin antennas. Electromagnetic simulation techniques can be used for practical dipole thicknesses, but no theory is available to benchmark the results of the numerical experiments. The feed modeling remains a long standing problem in terms of accurately matching the complete impedance to physical experimental results. To make a theoretical start on the problem, the induced EMF method with finite feed gap is solved here and the impedance of the thin dipole is presented. The effect of feed gap size for the finite length wire, e.g. the dipole antenna, has not been studied before. From the induced EMF method, the lossless, thin dipole with finite gap turns out to have an extremely wide bandwidth when terminated with 50 or 75 ohms, a new and interesting result in antenna theory.**

I. Introduction

A. The Input Region at the Terminals of an Antenna

Schelkunoff [1] seems to have been the first to address the ‘input region’ of a dipole - the region of transition between the feed line and the antenna. The detail of this region has a large impact on the reactive component of the input impedance or admittance. Schelkuoff offers little elaboration on the input region, but it has since become well established that the feed configuration affects the input impedance of most antennas e.g., [2], [3], including the dipole. The feed configuration in turn affects the modelled system performance because modelling errors in the impedance calculation cause lost power through errors in an implemented impedance matching. Because of the inaccurate input region of the models used by theory and simulation, the practical design of dipoles (and other elemental antennas), normally sees theoretical considerations discarded in favour of a cut-and-try approach. This motivates a further look at the input region of the basic dipole, and here we take an antenna-theoretic approach, and compare with the results from existing solution techniques.

In a practical situation, there is a gap between the dipole feed terminals, and these are connected directly to a transmission line. The transmission line detail is not addressed here except for the following comments. A pair of conductors forming a free space transmission line at right angles to the dipole can be used, and this configuration has traditionally been used for practical reference dipoles. Such a transmission line is restricted to a characteristic impedance of more than about a hundred ohms. Consequently a more likely architecture is a balun built-in to the input region and connected to a coax feed. The coax can be at right angles to the dipole. An alternative is a sleeve balun but this does not readily allow for thin conductors or a feed gap.

For physical measurements, the equivalent monopole on a groundplane is more convenient than the dipole. The monopole allows a direct coaxial feed through the groundplane or else connection to a planar transmission line on the groundplane. A very large groundplane is required if the impedance is to be compared with meaningful accuracy to theoretical dipole results. The monopole feed can be modeled either by an impressed current in the wire at, or close to, the groundplane position, or by a magnetic frill at the groundplane position with a physical size associated with the coaxial conductor spacing. Unfortunately the finite feed gap arrangement does not seem possible for the monopole configuration. Finally, the dipole cross section is circular corresponding to a wire construction.

There is a capacitance arising from the feed gap, classically given by $\epsilon\frac{A}{\delta}$, where the permittivity in the gap is ϵ, the area of the conductors is A, and δ is the gap width. The reactance component from this capacitor changes with frequency of course, and the infinitesimal gap size is problematic in the sense that the capacitance becomes infinite. However, this ca-

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pacitance problem can be averted if we have thin dipoles with a relatively large gap, i.e., when $\delta >> a$ where $a = \sqrt{A/2\pi}$ is the radius of the wire; and this is indeed the configuration of interest here. Similarly, local effects from the truncated open ends of the dipole are ignored.

B. Solution Techniques for the Impedance

The readily available methods for dipole analysis are: the induced EMF method, elaborated below; the wave structure method [4]; numerical techniques such as the method of moments (MOM) [5]; finite-difference time-domain (FDTD), e.g., [6], including variations such as CST; and finite element methods (FEM) [7] including HFSS; and finally, physical measurements.

The wave structure method is based on solutions for the waves on the semi-infinite wire, and as such it does not account for the input region. The impedance results seem to be reasonable only for very short dipoles, meaning lengths less than about 0.4λ.

The MOM finds the current distribution rather than assuming its form, but most results demonstrate that for thin wires, this current is very close to the sinusoidal form assumed in the induced EMF method. Discretizing the wire to an increasing number of smaller segments, does not necessarily bring greater accuracy, in general.

The FDTD method discretizes the time and space evolution of the fields from Maxwell's equations, and as such it is sensitive to numerically large ranges of structural dimensions. In particular, a very thin dipole (relative to its length) creates such dynamic range related problems.

Physical measurement is another approach to find the dipole impedance and is in some ways the ultimate test of accuracy of the models and solutions. But de-embedding the practical factors such as the balun used to properly balanced feed of the dipole, the limitations in determining exactly the thin wire lengths, or the need for a very large groundplane for the monopole configuration, all act to compromise the accuracy of physical measurements.

In Table I, some established methods for the impedance and the associated ranges of valid dipole length and thickness are summarized. The table lays out the dimensional limitation of each method for impedance accuracy, while l and a are the total dipole length and radius, respectively.

TABLE I
RANGE FOR GOOD RESULTS FROM METHODS OF DIPOLE IMPEDANCE, IN TERMS OF DIPOLE LENGTH l AND RADIUS a.

measurement methods	**Range of l**	**Range of a**
induced EMF method	$l \leq 0.8\lambda$	$a \leq 10^{-2}\lambda$
wave structure method	$l \leq 0.4\lambda$	$a \leq 10^{-7}\lambda$
MOM (WIPL)	any l	$a \leq 10^{-3}\lambda$
FDTD (CST)	any l	$10^{-2}\lambda \leq a$
FEM (HFSS)	any l	$10^{-3}\lambda \leq a$

Besides the dimension limitations, there are other shortcomings in each of these packages. Perhaps the most important challenge is the definition of the excitation in the finite gap. For example in WIPL-D the excitation configuration is a metallic apex [8] while in CST the excitation uses a variety of configurations (ports) for a voltage excitation. In any event, there is insufficient information about the configurations, assumptions and algorithms supplied with most of commercial EM solvers, to be able to comment on any expected shortcomings.

II. INDUCED EMF METHOD FOR FINITE GAP

The induced EMF method was originally presented by Brillouin [9]. It has been shown that the form of the induced EMF equation is almost exact [10]. The formulation does not feature an explicit feed model (although it can be implied by the formulation), and requires a current distribution, and uses this distribution directly to calculate the input impedance. The current is usually assumed to have a sinusoidal form (see below), and this assumption holds well for short dipoles. The assumption can be seen to break down for a full wavelength dipole - here the sinusoidal form of the feed current would be zero, and the impedance (voltage over current) would become infinite, which does not happen in practice.

The end effect of the dipole is neglected. For a solid conductor, current will flow across the end of the wires, and neglecting this current introduces an error of the order of ka, e.g., [11]. Since a is assumed to be small ($a = 10^{-4}\lambda$), the error will be in the order of a few times 10^{-4}. This is negligible in the context of the impact of other assumptions of the induced EMF method. Similarly, we do not account for any circumferential currents. In the context of theoretical excitations, there is is no mechanism for exciting the circumferential currents as long as the feed is azimuthally symmetric. Even in practice, for thin wire dipoles, the circumferential current, if present at all, has a negligible impact.

We also assume that the dipole is lossless. However, using good conductors such as silver and copper and even stainless steel does not affect the results much [12] at low frequencies. For example, for 1GHz, the ohmic resistance is less than 10Ω for $a = 10^{-4}\lambda$ and is less than 1Ω for $a = 10^{-3}\lambda$. At higher frequencies like 60GHz, gold would typically be used, and for $a = 10^{-3}\lambda$ the ohmic resistance is a couple of ohms, but for thinner wires of $a = 10^{-4}\lambda$ the ohmic resistance approaches the same order as radiation resistance [12].

For a dipole along the z axis, and wire radius of a, the induced EMF method requires the tangential z-directed electric field on the dipole surface $E_z(a, z)$. The excitation can be by magnetic frill or infinitesimal voltage gap or current element. Traditionally, a centre current element is used and so we use such an excitation here to offer consistent results with previous emf method solutions, i.e., for when there is no gap. We note that such an excitation current has no physical support in the context of a dipole with a central gap. A magnetic frill would give similar excitation, but again there is no physical structure for supporting the frill. These issues are typical in discussions of modelling the feed region. The

current excitation is given by, e.g., [13] as follows:

$$E_z = -j\frac{\eta I_o}{4\pi}\left[\frac{e^{-jkR_1}}{R_1} + \frac{e^{-jkR_2}}{R_2} - 2\cos(\frac{kl}{2})\frac{e^{-jkr}}{r}\right], \quad (1)$$

where R_1, R_2 and r, are the distances of the observation point to the both ends of the dipole and to the origin, respectively. μ is the permeability, k is the wave constant and I_0 is the excitation current. The current wave is modeled here as traveling at the speed of light, but the actual current velocity is a bit less than this. This velocity inaccuracy has little impact as long as the dipole length is short.
For a finite gap, the input impedance is then given by

$$Z_m = -\frac{1}{I_m}[\int_{-\frac{l}{2}}^{-\frac{\delta}{2}} sin\left[k(\frac{l}{2}+z^{'})\right] E_z(\rho=a, z=z^{'})dz^{'} + \int_{+\frac{\delta}{2}}^{+\frac{l}{2}} sin\left[k(\frac{l}{2}-z^{'})\right] E_z(\rho=a, z=z^{'})\ dz^{'}] \quad (2)$$

where δ is the distance between two arms of the dipole (gap size) and l is the total length of the dipole, $Z_m = R_m + jX_m$, and the input resistance R_m and reactance X_m are referred to I_m, the current maximum of the assumed sinusoidal distribution, rather than to the feed point current. The input resistance and input reactance at the input terminal are given by e.g., [13],

$$R_{in} = \frac{R_m}{sin^2(\frac{kl}{2})}, \quad X_{in} = \frac{X_m}{sin^2(\frac{kl}{2})}. \quad (3)$$

For a zero gap dipole, the reactance is a function of dipole thickness, but when the dipole length is a half-wavelength (or multiples of a half wavelength), the reactance of this expression is independent of the dipole thickness. Equation (2) with the embedded E_z cannot be solved analytically and in fact its numerical calculation has accuracy pitfalls. Here we have carefully used numerical integration methods from [14].

The input resistance and reactance of a dipole, with radius $a = 10^{-4}\lambda$ and various gap sizes, are depicted in Fig. 1.
The results show that for the range $0 < \delta < 10^{-2}\lambda$ there is no noticeable change in the resistance. But when δ is increased to 10^{-1} the change in resistance finally becomes noticeable and a sharp drop of about 20Ω happens at $l = \frac{\lambda}{2}$, and even more for a longer dipole. On the other hand the reactance is sensitive to the gap size and this rapid change governs the antenna bandwidth. When the gap becomes larger, the reactance curve tends to be flatter, implying a larger bandwidth. As the gap becomes sufficiently large (here for $\frac{\delta}{\lambda} = 0.1$), the resonance behaviour ceases in the sense that the reactance remains positive. Fig. 2 shows the same curves as an admittance. Here we see that the conductance and susceptance are not so well-defined as the resistance, as expected from the current excitation. The resonance behaviour is lost for the large gap case as noted from the reactance curves.

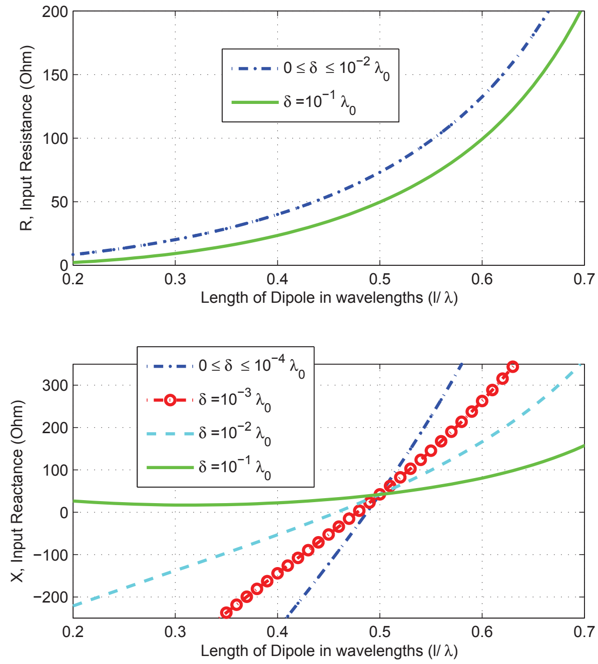

Fig. 1. Dipole impedance from the induced EMF method for varying feed gap and $a = 10^{-4}\lambda$. The solid lines are for a largest gap ($\delta = 10^{-1}\lambda$, almost 20% of the half-wavelength dipole), calculated using a sinusoidal current assumption.

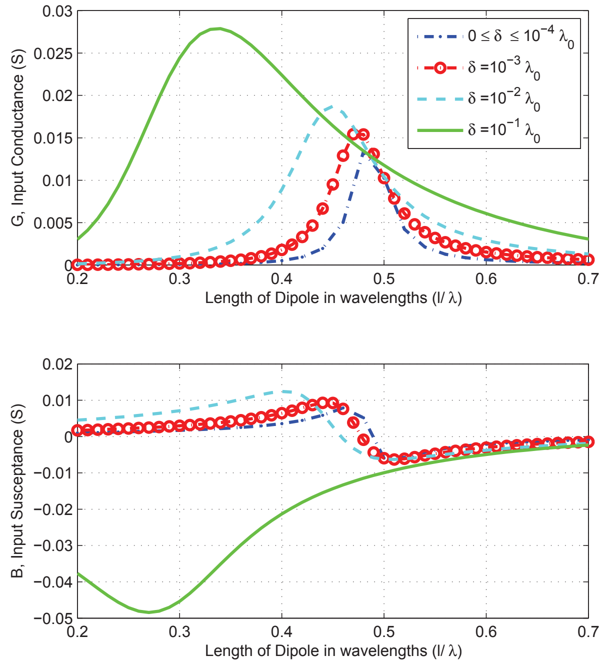

Fig. 2. Dipole admittance from the induced EMF method for varying feed gap.

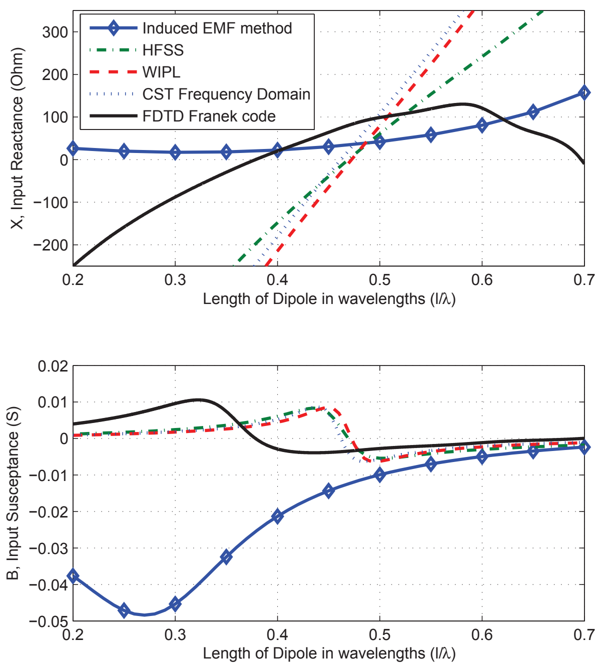

Fig. 3. Reactance and susceptance of a thin dipole of $\delta = 10^{-1}\lambda$ and $a = 10^{-4}\lambda$, from different numerical and theoretical methods.

III. Different Numerical Methods to Evaluate the Impedance of a Dipole

To study the gap effect, we have tried different methods:

- MOM [8]. Our impedance results were confirmed by WIPL-D expert customer support.
- CST. Microwave Studio This type of solver has problems with large dynamic range of dimensions, and uses adaptive meshing.
- HFSS. Our results were also confirmed by expert customer support
- FDTD. These results are from O. Franek [15]. Franek's rendition of FDTD allows only homogeneous mesh with one cell size throughout the domain. Since the δ gap must be equal to one cell, then the whole simulation has a cell size according to the desired gap.
- Induced EMF method. From this we present reference results for the impedance of dipoles with a finite gap.

The resistance curves from different methods are similar but we observe major differences in reactance results. In Fig. 3 the reactance and susceptance of the thin dipole of radius $a = 10^{-4}\lambda$ and gap size $\delta = 10^{-1}\lambda$ have been depicted. The closest results from other methods are those from Franek's FDTD [15]. Both FDTD results and the induced EMF results display a higher theoretical impedance bandwidth for a larger gap. This is most easily seen from the reactance curves.

IV. Conclusions

We have given reference results for the impedance of a lossless, thin dipole with a finite gap size, from the induced EMF method. Comparison with results from other techniques are included. To the best of our knowledge and ability, there is no single solution technique which is suitable for the impedance of the dipole over the full range of configurations of interest to the antenna theorist and practitioner. From our induced EMF results, which use a central current excitation, a larger gap size leads to a larger theoretical bandwidth of the dipole. Conventional practical wisdom suggests that a thicker dipole is better for a wider bandwidth, but the theoretical results show otherwise. In fact the bandwidth of the thin dipole is extremely large as long as there is a finite gap. Realizing an antenna with an idealized gap, and with the thin conductors, is a challenge. Moreover, the finite conductivity of a metal can compromise the useful bandwidth of the thin dipole. Nevertheless, the high bandwidth result is a promising pointer for further research.

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